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THE DETERMINATION OF UNSTEADY HEAT FLOW TO WALL FROM THE MEASUREMENTS OF SURFACE TEMPERATURE MADE WITH THIN FILM RESISTANCE THERMOMETERS

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Abstract—The paper derives equations for the calculation of unsteady heat flow to wall in cases where the time variation of temperature on the surface of the wall can be expressed by a power or trigonometric series. To make use of these equations, it is first necessary to establish reliably the square root of the product of heat conductivity, density and specific heat of the wall material, denoted by β, and to measure the time variation of the surface temperature rise with sufficient accuracy. Both tasks can be accomplished through the use of thin-film resistance thermometers consisting of a thin metallic layer deposited on an electrically insulating wall. The paper presents a detailed elaboration of the unsteady method of β measurement which facilitates determining this value with an error of about 3%. A method of measuring the surface temperature rise exceeding 3 degC with an error of about 2% is also described. The procedure discussed in the paper permits the determination of unsteady heat flow to wall to be made with an error of approximately 4%.

NOMENCLATURE

A, B, coefficients;
c, specific heat;
C, condenser capacity;
d, wall thickness;
D, expression defined by equation (40);
E, electric voltage;
ΔE, potential difference;
f, majorant function;
F, area;
i, electric current;
I, integral;
K_{11}, expression defined by equation (31);
K_{15}, expression defined by equation (32);
m, scale;
n, number;
q, heat flow;
Q, heat output;
r, a term of series $R(j\nu t)$;
$R(j\nu t)$, series defined by equation (23);
R, electric resistance;
R_{01}, resistance defined by equation (51);
R_{02}, resistance defined by equation (68);
s, a term of series $S(j\nu t)$;
$S(j\nu t)$, series defined by equation (24);
t, time;
T, temperature rise;
U, circumference;
x, y, co-ordinates;
Z, expression defined by equation (82).

Greek symbols

α, temperature coefficient of resistance;
β, $= \sqrt{(\lambda\rho c)}$;
γ, temperature coefficient of thermal conductivity;
δ, relative deviation;
ϵ, relative setting of potentiometer;
ζ_0, expression defined by equation (45);
ζ, expression defined by equation (46);
ζ_1, expression defined by equation (72);
ζ_2, expression defined by equation (73);
η, shift of origin in direction of y-axis;
ϑ, a variable;
$\varkappa$, limit absolute error;
$\bar{\varkappa}$, limit relative error;
λ, thermal conductivity;
Λ, heat-transfer coefficient defined by equation (93);
μ, coefficient of transverse heat flow effect;
ν, frequency;

ξ, a variable;
ρ, density;
σ, correction factor according to equation (69);
τ, period;
ϕ, expression defined by equation (56);
ψ, coefficient defined by equation (96);
Ω, expression defined by equation (37).

The SI system of units [kg of mass, s, m, °C, A] is used for all quantities.

INTRODUCTION

As EVIDENCED by a large number of treatises published on the subject in recent years, the determination of the heat flow to wall has been brought into the foreground of interest in research studies concerned with unsteady heat transfer. This is quite understandable because the heat flow usually expresses directly the losses of thermal energy and specifies, in conjunction with the surface temperature and the temperature of the medium, the local values of the heat-transfer coefficient.

Considerable attention has recently been accorded to the elaboration of the methods of measurement of rapidly variable surface temperatures with thin-film resistance thermometers (see, for example, [1–4]) gauges that consist of a thin metallic film deposited on an electrically insulating wall. Such thermometers record, almost without distortion [5] very fast (up to 1 degC/μs) variations of temperature of the surface on which they are produced. As pointed out by several investigators (see [1, 5–7]) this property can be utilized to advantage in the determination of unsteady heat flow to wall, particularly in connection with the shock tube technique.

The object of the presentation that follows is to refine further the methods suggested by those authors and to promote their application.

DEPENDENCE OF HEAT FLOW TO WALL OF A HALF-SPACE ON TIME VARIATION OF SURFACE TEMPERATURE

The conditions of experiments in which unsteady heat flows are studied, quite frequently approach those of heat conduction through a half-space whose thermophysical properties are independent of temperature and the time variation of the surface temperature rise is known.

The temperature rise of the half-space is described by the relation [8]:

$$T(x;t) = \frac{x}{2\sqrt{\pi}}\sqrt{\left(\frac{\rho c}{\lambda}\right)}\int_0^t \frac{T(0;t-\vartheta)}{\vartheta^{3/2}} \exp\left(-\frac{x^2}{4\vartheta}\frac{\rho c}{\lambda}\right) d\vartheta \quad (1)$$

where t denotes the time from the instant of $T(x;(0 = 0$, x is the co-ordinate in the direction of the normal to the surface, the variation of whose temperature rise $T(0;t)$ is known, and λ, ρ and c are respectively the thermal conductivity, density and specific heat, of the half-space. The heat flow through the surface of the half-space is given by equation

$$q = -\lambda \lim_{x\to 0} \frac{\partial T(x;t)}{\partial x}. \quad (2)$$

Equations (1) and (2) are solved on digital computers, graphically [2] or by means of analogue networks [9]. But very frequently the temperature variation $T(0;t)$ can be approximated by a power or trigonometric series; in such cases, one may apply the analytic solution outlined in the sections that follow.

When the temperature variation $T(0;t)$ is expressed by a power series,

$$T(0;t) = \sum_{j=0}^{n} A_j t^j \qquad (j = 0, 1, 2, \ldots) \quad (3)$$

it follows that

$$T(0;t-\vartheta) = \sum_{k=0}^{n} \bar{A}_k \vartheta^k \quad (k = 0, 1, 2, \ldots) \quad (4)$$

where

$$\bar{A}_k = \sum_{j=k}^{n} (-1)^k \binom{j}{k} A_j t^{j-k}. \quad (5)$$

Introducing relation (4) in equation (1) and substituting

$$\xi = \frac{x^2}{4\vartheta}\frac{\rho c}{\lambda} \quad (6)$$

we obtain

$$T(x;\,t) = \frac{1}{\sqrt{\pi}} \sum_{k=0}^{n} \bar{A}_k \left(\frac{x^2}{4}\,\frac{\rho c}{\lambda}\right)^k \int_{(x^2\rho c/4t\lambda)}^{\infty} \xi^{-k-\frac{1}{2}} \exp(-\xi)\, \mathrm{d}\xi. \qquad (7)$$

The function in the integrand can be integrated and the respective indefinite integral obtained by integration by parts in the form

$$\int \xi^{-k-\frac{1}{2}} \exp(-\xi)\,\mathrm{d}\xi = \exp(-\xi) \sum_{l=1}^{\infty} \frac{(-1)^l \xi^{-k-\frac{1}{2}+l}}{\binom{k-\frac{1}{2}}{l} l!} \quad (l = 1, 2, \ldots)$$

Since the series on the right-hand side of the equation is an alternating one and it holds that

$$\lim_{\xi\to\infty} \frac{(-1)^l \xi^{-k-\frac{1}{2}+l}}{\binom{k-\frac{1}{2}}{l} l! \exp \xi} = \theta,$$

the integral in equation (7) is convergent and the equation takes the form of

$$T(x;\,t) = -\frac{1}{\sqrt{\pi}} \sum_{k=0}^{n} \bar{A}_k \exp\left(-\frac{x^2}{4t}\,\frac{\rho c}{\lambda}\right) \sum_{l=1}^{\infty} \frac{(-1)^l \{(x/2)[\sqrt{(\rho c/\lambda)}]\}^{2l-1}}{\binom{k-\frac{1}{2}}{l} l!}\, t^{k+\frac{1}{2}-l} \qquad (8)$$

As the partial derivative of expression (8) with respect to x and its limit for x approaching zero indicate, it holds that

$$\lim_{x\to 0} \frac{\partial T(x;\,t)}{\partial x} = \sqrt{\left(\frac{\rho c}{\pi\lambda}\right)} \sum_{k=0}^{n} \frac{\bar{A}_k}{2k-1}\, t^{k-\frac{1}{2}}. \qquad (9)$$

Finally it follows from equations (2), (5) and (9) that

$$q = -\sqrt{\left(\frac{\lambda\rho c}{\pi t}\right)} \left\{ \sum_{k=0}^{n} \frac{(-1)^k}{2k-1} \sum_{j=k}^{n} A_j \binom{j}{k} t^j \right\} \qquad (10)$$

If $n = 0$, it is a temperature jump that is involved on the surface of the half-space with value

$$A_0 = T.$$

For such a case equation (10) yields the well-known relation

$$q = \sqrt{\left(\frac{\lambda\rho c}{\pi t}\right)}\, T. \qquad (11)$$

If $n > 0$, function $q(t)$ is more complicated; thus, for example, for $n = 4$ we get

$$q = \sqrt{\left(\frac{\lambda\rho c}{\pi t}\right)} \left(A_0 + 2A_1 t + \tfrac{8}{3} A_2 t^2 + \tfrac{16}{5} A_3 t^3 + \tfrac{128}{35} A_4 t^4\right). \qquad (12)$$

When function $T(0;\,t)$ can be expressed by a trigonometric series

$$T(0;\,t) = A_0 + \sum_{j=1}^{n} (A_j \cos j\nu t + B_j \sin j\nu t) \quad (j = 1, 2, \ldots) \qquad (13)$$

where

$$\nu = \frac{2\pi}{\tau} \qquad (14)$$

τ being the period of function $T(0;\,t)$, it holds that

$$T(0;\,t-\vartheta) = A_0 + \sum_{j=1}^{n} (\bar{A}_j \cos j\nu\vartheta + \bar{B}_j \sin j\nu\vartheta) \qquad (15)$$

where

$$\left.\begin{aligned} \bar{A}_j &= A_j \cos j\nu t + B_j \sin j\nu t \\ \bar{B}_j &= A_j \sin j\nu t - B_j \cos j\nu t \end{aligned}\right\} \qquad (16)$$

It follows from equations (1) and (15) that

$$\left.\begin{aligned} T(x;\,t) = {} & A_0 \frac{x}{2} \sqrt{\left(\frac{\rho c}{\pi\lambda}\right)} \int_0^t \vartheta^{-3/2} \exp\left(-\frac{x^2}{4\vartheta}\,\frac{\rho c}{\lambda}\right) \mathrm{d}\vartheta \\ & + \frac{x}{2} \sqrt{\left(\frac{\rho c}{\pi\lambda}\right)} \sum_{j=1}^{n} \bar{A}_j \int_0^t \frac{\cos j\nu\vartheta}{\vartheta^{3/2}} \exp\left(-\frac{x^2}{4\vartheta}\,\frac{\rho c}{\lambda}\right) \mathrm{d}\vartheta \\ & + \frac{x}{2} \sqrt{\left(\frac{\rho c}{\pi\lambda}\right)} \sum_{j=1}^{n} \bar{B}_j \int_0^t \frac{\sin j\nu\vartheta}{\vartheta^{3/2}} \exp\left(-\frac{x^2}{4\vartheta}\,\frac{\rho c}{\lambda}\right) \mathrm{d}\vartheta. \end{aligned}\right\} \qquad (17)$$

Denoting successively by I_1, I_2, I_3 the terms on the right-hand side of the equation, we obtain from relations (2) and (17)

$$q = -\lambda\left(\lim_{x\to 0}\frac{\partial I_1}{\partial x} + \lim_{x\to 0}\frac{\partial I_2}{\partial x} + \lim_{x\to 0}\frac{\partial I_3}{\partial x}\right). \quad (18)$$

According to the preceding exposition

$$\lim_{x\to 0}\frac{\partial I_1}{\partial x} = -A_0\sqrt{\left(\frac{\rho c}{\pi\lambda t}\right)}. \quad (19)$$

It is furthermore necessary to evaluate the following integrals

$$\left.\begin{aligned} I_{2j} &= \int_0^t \frac{\cos j\nu\vartheta}{\vartheta^{3/2}}\exp\left(-\frac{x^2}{4\vartheta}\frac{\rho c}{\lambda}\right)\mathrm{d}\vartheta \\ I_{3j} &= \int_0^t \frac{\sin j\nu\vartheta}{\vartheta^{3/2}}\exp\left(-\frac{x^2}{4\vartheta}\frac{\rho c}{\lambda}\right)\mathrm{d}\vartheta \end{aligned}\right\} \quad (20)$$

The functions in the integrands of equations (20) can be integrated and have the following majorant function

$$f = \vartheta^{-3/2}\exp\left(-\frac{x^2}{4\vartheta}\frac{\rho c}{\lambda}\right).$$

Since the integral in equation (7) is convergent, integrals I_{2j}, I_{3j}, are also convergent. We first evaluate integrals

$$I_{21} = \int\frac{\cos j\nu\vartheta}{\vartheta^{3/2}}\,\mathrm{d}\vartheta;\quad I_{31} = \int\frac{\sin j\nu\vartheta}{\vartheta^{3/2}}\,\mathrm{d}\vartheta. \quad (21)$$

Irrespective of the integration constant, the method by parts results in

$$\left.\begin{aligned} I_{21} = &-\frac{2}{\sqrt{\vartheta}}\left[\cos j\nu\vartheta\sum_{k=0}^{\infty}\frac{(-1)^k(j\nu\vartheta)^{2k}}{\binom{2k-\frac{1}{2}}{2k}(2k)!}\right. \\ &\left.+\,2j\nu\vartheta\sin j\nu\vartheta\sum_{k=0}^{\infty}\frac{(-1)^k(j\nu\vartheta)^{2k}}{\binom{2k+\frac{1}{2}}{2k}(2k)!}\right] \\ I_{31} = &-\frac{2}{\sqrt{\vartheta}}\left[\sin j\nu\vartheta\sum_{k=0}^{\infty}\frac{(-1)^k(j\nu\vartheta)^{2k}}{\binom{2k-\frac{1}{2}}{2k}(2k)!}\right. \\ &\left.-\,2j\nu\vartheta\cos j\nu\vartheta\sum_{k=0}^{\infty}\frac{(-1)^k(j\nu\vartheta)^{2k}}{\binom{2k+\frac{1}{2}}{2k}(2k)!}\right] \\ &(k = 0, 1, 2, \ldots) \end{aligned}\right\} \quad (22)$$

Furthermore letting

$$R(j\nu t) = \sum_{k=0}^{\infty}\frac{(-1)^k(j\nu t)^{2k}}{\binom{2k-\frac{1}{2}}{2k}(2k)!} \quad (23)$$

$$S(j\nu t) = 2j\nu t\sum_{k=0}^{\infty}\frac{(-1)^k(j\nu t)^{2k}}{\binom{2k+\frac{1}{2}}{2k}(2k)!}, \quad (24)$$

we can, in view of the convergence of integrals I_{2j}, I_{3j} write their solution obtained by the method by parts, in the form

$$\left.\begin{aligned} I_{2j} = &-\frac{2}{\sqrt{t}}\exp\left(-\frac{x^2}{4t}\frac{\rho c}{\lambda}\right) \\ &[R(j\nu t)\cos j\nu t + S(j\nu t)\sin j\nu t] \\ &-\frac{x^2}{4}\frac{\rho c}{\lambda}\int_0^t\frac{I_{21}}{\vartheta^2}\exp\left(-\frac{x^2}{4\vartheta}\frac{\rho c}{\lambda}\right)\mathrm{d}\vartheta \\ I_{3j} = &-\frac{2}{\sqrt{t}}\exp\left(-\frac{x^2}{4t}\frac{\rho c}{\lambda}\right) \\ &[R(j\nu t)\sin j\nu t - S(j\nu t)\cos j\nu t] \\ &-\frac{x^2}{4}\frac{\rho c}{\lambda}\int_0^t\frac{I_{31}}{\vartheta^2}\exp\left(-\frac{x^2}{4\vartheta}\frac{\rho c}{\lambda}\right)\mathrm{d}\vartheta. \end{aligned}\right\} \quad (25)$$

The question now is the convergence of series $R(j\nu t)$ and $S(j\nu t)$. Denoting by r, s the terms of the respective series, we get from equations (23) and (24).

$$\begin{aligned} \left|\frac{r_{k+1}}{r_k}\right| &= \frac{(j\nu t)^2}{(2k+\frac{3}{2})(2k+\frac{1}{2})} \\ \left|\frac{s_{k+1}}{s_k}\right| &= \frac{(j\nu t)^2}{(2k+\frac{5}{2})(2k+\frac{3}{2})} \end{aligned} \quad (26)$$

Since the absolute value of the ratio of successive terms of the two alternating series decreases with increasing k and for k increasing beyond all bounds approaches zero, series $R(j\nu t)$ and

$S(j\nu t)$ converge. Consequently, the integrals in equations (25) also converge. It follows from relations (17) and (25) that

$$\left.\begin{aligned} I_2 &= -\frac{x}{2}\sqrt{\left(\frac{\rho c}{\pi\lambda t}\right)}\exp\left(-\frac{x^2}{4t}\frac{\rho c}{\lambda}\right) \\ &\sum_{j=1}^{n}\bar{A}_j\left\{[R(j\nu t)\cos j\nu t + S(j\nu t)\sin j\nu t]\right. \\ &\left.-\frac{x^3}{8}\sqrt{\left(\frac{\rho^3c^3}{\pi\lambda^3}\right)}\int_{\Theta}^{t}\frac{I_{21}}{\vartheta^2}\exp\left(-\frac{x^2}{4\vartheta}\frac{\rho c}{\lambda}\right)d\vartheta\right\} \\ I_3 &= -\frac{x}{2}\sqrt{\left(\frac{\rho c}{\pi\lambda t}\right)}\exp\left(-\frac{x^2}{4t}\frac{\rho c}{\lambda}\right) \\ &\sum_{j=1}^{n}\bar{B}_j\left\{[R(j\nu t)\sin j\nu t - S(j\nu t)\cos j\nu t]\right. \\ &\left.\left(-\frac{x^3}{8}\sqrt{\frac{\rho^3c^3}{\pi\lambda^3}}\right)\int_{\Theta}^{t}\frac{I_{31}}{\vartheta^2}\exp\left(-\frac{x^2}{4\vartheta}\frac{\rho c}{\lambda}\right)d\vartheta\right\}. \end{aligned}\right\} \quad (27)$$

Carrying out in equations (27) partial derivatives with respect to x and limits of these derivatives for x approaching zero, gives

$$\left.\begin{aligned} \lim_{x\to\Theta}\frac{\partial I_2}{\partial x} &= -\sqrt{\left(\frac{\rho c}{\pi\lambda t}\right)}\sum_{j=1}^{n}\bar{A}_j \\ &[R(j\nu t)\cos j\nu t + S(j\nu t)\sin j\nu t] \\ \lim_{x\to\Theta}\frac{\partial I_3}{\partial x} &= -\sqrt{\left(\frac{\rho c}{\pi\lambda t}\right)}\sum_{j=1}^{n}\bar{B}_j \\ &[R(j\nu t)\sin j\nu t - S(j\nu t)\cos j\nu t]. \end{aligned}\right\} \quad (28)$$

Introducing finally relations (16), (19), (28) into equation (18), we obtain the solution in the form of

$$q = \sqrt{\left(\frac{\lambda\rho c}{\pi t}\right)}\,\{A_0 + \sum_{j=1}^{n}[A_j\,R(j\nu t) + B_j\,S(j\nu t)]\}. \quad (29)$$

Series $R(j\nu t)$ and $S(j\nu t)$ are best evaluated with the aid of relations (26). The results thus arrived at are summarized in Table 1 and Fig. 1.

The solutions presented in the foregoing remain, of course, valid only as long as the wall can be considered a half-space, i.e. as long as

$$t \ll \frac{\rho c}{\lambda}d^2 \quad (30)$$

where d is the thickness of the wall on which the time variation of surface temperature is to be studied. Times up to 1 s, quite satisfactory especially for shock tube experiments, are usually acceptable for the backing of the thin-film thermometers.

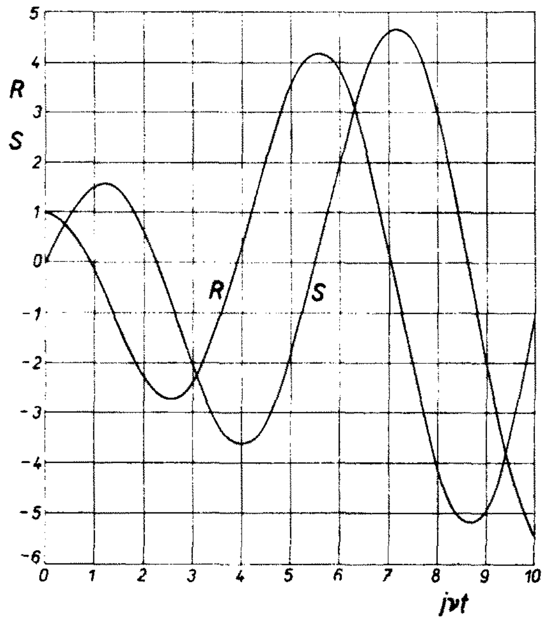

FIG. 1. Curves of function $R(j\nu t)$ and $S(j\nu t)$.

MEASURING THE VALUE OF $\sqrt{(\lambda\rho c)}$ OF THE WALL

The first prerequisite for the application of the equations derived in the preceding section, is reliable knowledge of the product of thermal conductivity, density and specific heat of the wall material, which appears in them. These properties can, of course, be either estimated on the basis of data reported in literature [4], or determined through the measurement of samples of the wall material. The first alternative might lead to considerable errors, the second is both laborious and incapable of expressing the effect

Table 1. *Values of functions* $R(j\nu t)$ and $S(j\nu t)$

$j\nu t$	$R(j\nu t)$	$S(j\nu t)$	$j\nu t$	$R(j\nu t)$	$S(j\nu t)$
0	1·00	0	5·2	3·89	−1·27
0·2	0·95	0·40	5·4	4·12	−0·49
0·4	0·79	0·77	5·6	4·19	0·35
0·6	0·54	1·09	5·8	4·10	1·18
0·8	0·21	1·34	6·0	3·82	2·03
1·0	−0·19	1·50	6·2	3·39	2·78
1·2	−0·62	1·56	6·4	2·80	3·46
1·4	−1·07	1·51	6·6	2·07	3·99
1·6	−1·51	1·34	6·8	1·24	4·39
1·8	−1·92	1·08	7·0	0·34	4·61
2·0	−2·26	0·70	7·2	−0·61	4·65
2·2	−2·52	0·25	7·4	−1·56	4·48
2·4	−2·67	−0·27	7·6	−2·47	4·13
2·6	−2·71	−0·84	7·8	−3·30	3·65
2·8	−2·62	−1·41	8·0	−4·01	2·91
3·0	−2·40	−1·97	8·2	−4·58	2·08
3·2	−2·06	−2·49	8·4	−4·98	1·18
3·4	−1·60	−2·94	8·6	−5·19	0·15
3·6	−1·04	−3·30	8·8	−5·18	−0·90
3·8	−0·40	−3·53	9·0	−4·97	−1·94
4·0	0·29	−3·64	9·2	−4·55	−2·91
4·2	1·01	−3·60	9·4	−3·93	−3·78
4·4	1·72	−3·42	9·6	−3·14	−4·55
4·6	2·40	−3·07	9·8	−2·21	−5·12
4·8	3·00	−2·58	10·0	−1·17	−5·56
5·0	3·51	−1·98			

of local inhomogeneities. These are the reasons for suggesting [2] the unsteady method of measurement in order to determine directly the value of $\sqrt{(\lambda\rho c)}$; it involves the construction on the wall of a thin-film resistance thermometer with approximately constant, electrically induced heat flow to the wall. The value of $\sqrt{(\lambda\rho c)}$ can then be determined from the time variation of the surface temperature rise. So long as a resistance film can be produced on the wall, the method is advantageous inasmuch as it can also be applied in cases where the samples of the wall material are not available. The accuracy of the method has, however, been quite low so far: on the basis of data published in [1] and [2] one can estimate the limit error of $\sqrt{(\lambda\rho c)}$ as ± 6–$8\,\%$. The accuracy of the method can be enhanced by detailed elaboration.

Figure 2 gives the schematic diagram of the measuring equipment as reported by Henshall and Schultz [2]. The thin-film thermometer R is connected in a bridge first energized through switch 1 from a low-voltage battery and indicated by a galvanometer. The bridge is balanced with potentiometer R_7, connected through switch 1 to an oscilloscope and through switch 2 to higher d.c. condenser voltage E. The oscilloscope screen displays the time variation of potential difference ΔE between the measuring nodal points of the bridge in time t.

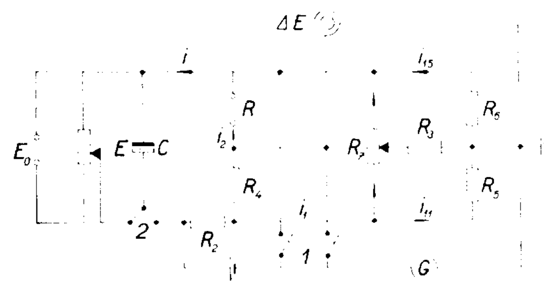

FIG. 2. Bridge for measuring values of β. $R_2 = 36\,\Omega$; $R_3 = 160\,\Omega$; $R_4 = 47\,\Omega$; $R_5 = 30\,\Omega$; $R_6 = 43\,\Omega$; $R_7 = 68\,\Omega$.

An analysis of the electric conditions of the bridge will be made on the assumption of no current passing through the indicating instrument and negligible resistances of the thermometer leads. Writing

$$K_{11} = \frac{\epsilon\left(1+\frac{R_6}{R_3}\right)+\left(1-\epsilon\right)\left(1+\epsilon\frac{R_7}{R_3}\right)}{\epsilon+\frac{R_5}{R_7}+\left(\frac{R_6}{R_7}+1-\epsilon\right)\left(1+\frac{R_5}{R_3}+\epsilon\frac{R_7}{R_3}\right)} \quad (31)$$

$$K_{15} = K_{11}\left(1+\frac{R_5}{R_3}+\epsilon\frac{R_7}{R_3}\right)-\epsilon\frac{R_7}{R_3} \quad (32)$$

where ϵ is the relative setting of potentiometer R_7, we obtain (Fig. 2)

$$i_{11} = K_{11}i_1; \quad i_{15} = K_{15}i_1 \quad (33)$$

and also

$$i = \frac{E}{\Omega R_2}\left(\frac{R_4}{R_5}+\frac{R}{R_5}+K_{11}+K_{15}\frac{R_6}{R_5}\right) \quad (34)$$

$$i_2 = \frac{E}{\Omega R_2}\left(K_{11}+K_{15}\frac{R_6}{R_5}\right) \quad (35)$$

$$\Delta E = \frac{E}{\Omega}\left(K_{11}\frac{R}{R_2}-K_{15}\frac{R_4R_6}{R_2R_5}\right) \quad (36)$$

where

$$\Omega = \frac{R_4}{R_5}+\frac{R}{R_5}+\left(1+\frac{R_4}{R_2}+\frac{R}{R_2}\right)\left(K_{11}+K_{15}\frac{R_6}{R_5}\right). \quad (37)$$

For a balanced bridge $R = R_0$, $\Delta E = 0$, and equation (34) results in

$$K_{11} = K_{15}\frac{R_4R_6}{R_0R_5}. \quad (38)$$

Introducing relations (31), (32) to equation (38), we obtain

$$\frac{R_3}{R_7}+\frac{R_5}{DR_7}+\epsilon\left(\frac{R_6+R_7}{R_7}-\frac{R_5+R_6}{DR_7}\right)-\epsilon^2 = 0 \quad (39)$$

where

$$D = 1-\frac{R_0R_5}{R_4R_6} \quad (40)$$

whence follows the value of ϵ for the given R_0. Since ϵ can vary only within the interval of (0; 1), we also obtain from equations (39) and (40) the limits of values of R_0 which can be balanced by the bridge:

$$R_{0\max} = \left(1+\frac{R_5}{R_3}\right)\frac{R_6}{R_5}R_4;$$

$$R_{0\min} = 1+\frac{R_6/R_5}{1+(R_6/R_3)}R_4. \quad (41)$$

Over the relatively narrow temperature interval in which the resistance film is heated by electric pulse during the measurement, the film resistance may be expressed by relation

$$R = R_0(1+\alpha T) \quad (42)$$

where α is the temperature coefficient of resistance, R_0 the resistance of the film at temperature of the bridge balance, and T the respective temperature rise.

Equations (36), (38) and (42) result in

$$\Delta E = E\frac{R_0}{R_2}\frac{\alpha K_{11}T}{\Omega}. \quad (43)$$

The heat output produced in the film during the condenser discharge, is given by equation

$$Q = i_2^2R. \quad (44)$$

Letting

$$\zeta_0 = \left[\frac{R_0}{\Omega_0R_2}\left(K_{11}+K_{15}\frac{R_6}{R_5}\right)\right]^2 \quad (45)$$

$$\zeta = \frac{R_0}{\Omega_0R_5}\left(1+K_{11}\frac{R_5}{R_2}+K_{15}\frac{R_6}{R_2}\right) \quad (46)$$

where Ω_0 denotes the value of Ω for $R = R_0$, we obtain

$$Q = \frac{E^2}{R_0}\zeta_0\frac{1+\alpha T}{(1+\alpha\zeta T)^2}. \quad (47)$$

If the resistance of the film remained unvaried ($R = R_0$) during its heating, the corresponding heat output would assume the value of Q_0. Since $\alpha T \ll 1$, the relative change of output due to the heating of the film follows with sufficient accuracy from equation (47) in the form

$$\delta Q = \frac{Q-Q_0}{Q_0} \cong (1-2\zeta)\alpha T, \quad (48)$$

The effect of the change in film resistance on heat flow can to a considerable extent be compensated by the choice of ζ. If it is required that $\delta Q = \vartheta$, we obtain from equations (37), (46) and (48) the following condition

$$\frac{R_0 - R_4}{R_5} = \frac{K_{11} + K_{15}(R_6/R_5)}{1 + K_{11}(R_5/R_2) + K_{15}(R_6/R_2)}. \tag{49}$$

Evidently, $\delta Q = \vartheta$ can be attained at the given resistances of the bridge for a single value of R_0 only.

The heat output produced in the film, changes moreover because of a drop in voltage E during the condenser discharge. The condenser discharge is described by

$$i\,dt = -C\,dE. \tag{50}$$

Writing

$$R_{01} = \frac{R_2\Omega_0}{(R_4/R_5) + (R_0/R_5) + K_{11} + (K_{15}(R_6/R_5)} \tag{15}$$

we get from equations (34) and (51) approximately

$$i = \frac{E}{R_{01}}. \tag{52}$$

For the boundary condition of $E = E_0$ and for $t = \theta$, the solution of equations (50) and (52) takes the form of

$$\ln\frac{E}{E_0} = -\frac{t}{CR_{01}}. \tag{53}$$

Bearing in mind that the allowable drop in condenser voltage is very small in the course of the measurement, we can express with sufficient accuracy the relative change of voltage as

$$\delta E = \frac{E_0 - E}{E_0} \cong \frac{t}{CR_{01}}. \tag{54}$$

Essentially, heat from the resistance layer is transferred only by conduction to the wall and conduction to the ambient air. Natural convection cannot develop during the short times ($t < 1$ ms) of measurement, and the effect of radiation is negligible for the usual experimental arrangement [10]. Since the changes of heat output produced in the film can be made very small, the case under discussion can in the first approximation be considered that of heat conduction by two neighbouring half-spaces with a constant heat flow in the interface. However, the temperature rise can sometimes be so large that the dependence of thermal conductivity of the half-space on temperature must be taken into consideration. A solution of such a case was presented by Hartunian and Varvig [3]. According to these authors, the relation for the interface can be written as

$$\phi = 2q\sqrt{\left(\frac{\lambda_0 t}{\pi\rho_0 c_0}\right)} \tag{55}$$

where q is the heat flow to the half-space,

$$\phi = \int_{\vartheta}^{T} \lambda\,dT \tag{56}$$

and subscript ϑ refers to the thermophysical properties of the half-space at the temperature prior to the condenser discharge. Hartunian and Varvig [3] selected a logarithmic function for the purpose of expressing the dependence of $\lambda(T)$. But a linear function

$$\lambda = \lambda_0(1 + \gamma T) \tag{57}$$

satisfies the temperature intervals that come into consideration, just as well.

The values of coefficient γ of some of the relevant substances are listed in Table 2.

Table 2. Values of 1000γ [1/degC] *for temperature of* 300°K *according to data published in references* 3 *and* 11

	Reference 3	Reference 11
Air		2·32
Glass	2·28	1·56
Quartz	2·40	
Pyrex	5·50	1·24

From equations (55)–(57) we obtain

$$T\left(1 + \frac{\gamma}{2}T\right) = \frac{2}{\sqrt{\pi}}q\sqrt{\left(\frac{t}{\lambda_0\rho_0 c_0}\right)}. \tag{58}$$

Actually, however, the heat flow from the resistance film to the wall is not one-dimensional as assumed by equation (55), because of finite dimensions of the film: hence heat flows not only in the direction normal, but also parallel, to the surface.

Exact solution of cases similar to the one just mentioned, is very difficult, as pointed out, for example, by Bailey [12]; but the effect of transverse heat flow cannot be left out of our considerations without closer analysis. Since the transverse heat output is smaller in order of magnitude than the output flowing in the direction of the normal, we shall be satisfied with the following approximate solution: Express the heat conducted to the wall by equation

$$Q_s = \mu q F \tag{59}$$

where F is the geometric area of the resistance film, q the heat flow corresponding to the one-dimensional case, and μ the correction factor.

Heat Q_s is given by relation

$$Q_s = Q_x + Q_y \tag{60}$$

where, referring to Fig. 3,

$$Q_x = qF \tag{61}$$

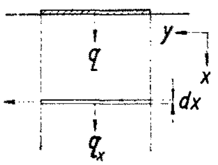

FIG. 3. Heat flows in wall.

The heat flow in the x-direction is given approximately [8] as

$$q_x = q \operatorname{erfc} \frac{x}{2\sqrt{[(\lambda_0/\rho_0 c_0)t]}} \tag{62}$$

assume that the heat flows in the direction of x and y (y being the cylindrical co-ordinate) are equal at a given distance from the surface, i.e.

$$q_y \cong q_x. \tag{63}$$

Such an assumption appears reasonable because the dimensions of the wall through which heat is conducted (thin-film thermometer) are of the same order of magnitude in the x- and y-direction. Heat conducted in the y-direction can be expressed as

$$Q_y = U \int_0^\infty q_y \, dx \tag{64}$$

where U is the circumference of the resistance film, and equations (59)–(64) result in

$$\mu = 1 + \frac{2}{\sqrt{\pi}} \frac{U}{F} \sqrt{\left(\frac{\lambda_0 t}{\rho_0 c_0}\right)}. \tag{65}$$

The total heat removed from the resistance film is given by relation

$$Q = Q_s + Q_v \tag{66}$$

where Q_v is the heat conducted to the ambient air. Since at the interface, the temperature rise of the air is identical with that of the wall, $Q_v \ll Q_s$ and the values of γ are of the same order of magnitude for air and wall, according to Table 2, we may, in view of equations (58), (59) and (66), write with sufficient accuracy that

$$Q = Q_s \left\{1 + \sqrt{\left[\frac{(\lambda_0 \rho_0 c_0)_v}{\lambda_0 \rho_0 c_0}\right]}\right\}. \tag{67}$$

Subscript v refers to the thermophysical properties of air.

Writing furthermore

$$R_{02} = R_0 \left(\frac{R_2}{R_0}\right)^3 \frac{\Omega_0^3}{K_{11}[K_{11} + K_{15}(R_6/R_5)]^2} \tag{68}$$

$$\sigma = 1 + \left[\frac{\gamma}{2} + (3\zeta - 1)\,\alpha\right] T + \sqrt{\left[\frac{(\lambda_0 \rho_0 c_0)_v}{\lambda_0 \rho_0 c_0}\right]} + \frac{2}{\sqrt{\pi}} \frac{U}{F} \sqrt{\left[\frac{\lambda_0 t}{\rho_0 c_0}\right]} \tag{69}$$

$$\beta = \sqrt{(\lambda_0 \rho_0 c_0)} \tag{70}$$

and assuming that the condenser voltage changes but little ($E \cong E_0$) during the measurement, the following expression is obtained with sufficient accuracy from equations (43), (45)–(47), (58), (59), (65) and (67)–(70):

$$\beta = \frac{2}{\sqrt{\pi}} \frac{\sqrt{t}}{\Delta E} \frac{E_0^3}{R_{02}} \frac{\alpha}{\sigma F}. \tag{71}$$

Letting finally

$$\zeta_1 = \frac{R_2}{R_0} \frac{\Omega_0}{K_{11}} \tag{72}$$

$$\zeta_2 = \frac{1}{K_{11}} \left(\frac{R_2}{R_5} + K_{11} + K_{15} \frac{R_6}{R_5}\right) \tag{73}$$

we can also ascertain the temperature rise T in time t following the beginning of the condenser discharge because equations (42), (43), (72) and (73) give

$$T = \frac{\Delta E}{\alpha E_0} \frac{\zeta_1}{1 - (\Delta E/E_0)\,\zeta_2}. \tag{74}$$

Time t in which the measurements are taken, is but a fraction of the time necessary for complete discharge of the condenser. The total temperature rise T_m of the film is therefore substantially larger than temperature rise T during the experiment. The value of T_m must not exceed a definite limit in order not to damage the film. Exact calculation of T_m is very complex because the voltage and hence also the heat produced in the film, undergo strong changes during the condenser discharge. An approximate calculation is carried out by considering constant production of heat Q_0 in the film taking place only for such a time t_m that the energy is the same as the actual one, i.e.

$$t_m = \frac{1}{Q_0} \int_0^\infty Q_t \, dt. \tag{75}$$

In view of equation (47), we may approximately put

$$Q_t = Q_0 \left(\frac{E}{E_0}\right)^2 \tag{76}$$

it then follows from relations (53), (54), (75) and (76) that

$$t_m = \frac{CR_{01}}{2} \cong \frac{t}{2\delta E}. \tag{77}$$

Since according to equations (71) and (74) it approximately holds that

$$\frac{T_m}{T} \cong \sqrt{\left(\frac{t_m}{t}\right)} \tag{78}$$

we get from expressions (74), (77) and (78) that

$$T_m = \frac{T}{\sqrt{(2\delta E)}} \cong \frac{\Delta E}{E_0} \frac{\zeta_1}{\alpha\sqrt{(2\delta E)}}. \tag{79}$$

The resistances of the bridge shown in Fig. 2 were determined so as to enable us to balance the basic resistance of the thin-film thermometer within the limits of $R_0 = 55$ to $80\ \Omega$ [relations (41)], to meet condition (49) for $R_0 \cong 67{\cdot}5\ \Omega$, and to use resistances of a standard series. The resultant resistances are given in Fig. 2, the characteristic functions of the bridge ascertained for these values, in Table 3. The main advantage lies in that the basic resistance R_0 of the thermometer matters very little.

As an analysis and the results of experiments indicate, it is convenient to choose $t = 500\ \mu$s, $\Delta E = 50$ mV, $\delta E = 0{\cdot}0025$ [10]. The corresponding values of E_0 and T_m at $R_0 = 67{\cdot}5\ \Omega$, $\sigma = 1$ are shown in Table 4 for limit values of α, β, F. It is evident that the necessary condenser voltages are readily attainable, and the maximum temperature rise of the film is quite acceptable.

According to relation (54), the condenser should have for the given values a capacity $C \cong 3000\ \mu$F which can economically be realized only through the use of electrolytic condensers. But such condensers have a relatively low bleeder resistance and it is, therefore, necessary

Table 3. Characteristic functions of the bridge shown in Fig. 2

$R_0(\Omega)$	$R_{01}(\Omega)$	$R_{02}(\Omega)$	ζ_1	ζ_2	ζ
55	61·2	10 880	9·77	4·50	0·46
57·5	61·8	10 600	9·70	4·57	0·47
60	62·2	10 410	9·66	4·65	0·48
62·5	62·5	10 270	9·63	4·73	0·49
65	62·8	10 240	9·64	4·83	0·50
67·5	63·0	10 220	9·65	4·92	0·51
70	63·0	10 310	9·70	5·04	0·52
72·5	63·1	10 450	9·77	5·16	0·53
75	63·0	10 630	9·85	5·29	0·54
77·5	63·0	10 840	9·94	5·42	0·55
80	62·9	11 090	10·00	5·56	0·56

Table 4. Necessary condenser voltage and maximum temperature rise of film

F (mm^2)	$\alpha . 10^3$ (1/degC)	β ($kg/s^{5/2}degC$)	E_0 (V)	T_m (degC)
2	0·7	700	35	282
		1500	44	219
	3	700	21	110
		1500	27	83
40	0·7	700	94	104
		1500	121	80
	3	700	58	39
		1500	75	30

to accomplish the switching of the condenser from the source to the bridge in a short time. A tilting mercury switch that effects the operation in 10–40 ms, with the condenser voltage falling less than 0·1%, has been found very useful in this respect. The accuracy of measurement is substantially enhanced because under such circumstances the voltage of the source can be taken as E_0.

The transient phenomena that arise immediately following the switching-on of the condenser, in particular the charging of the lead capacities, are the reason why the origin for the reading of the potential difference ΔE is usually not known exactly. The equations of the displayed oscilloscope traces is in the form

$$\Delta E = m_E(y + \eta) \tag{80}$$

$$t = m_t x \tag{81}$$

where x, y are the oscillogram co-ordinates, m_t, m_E are the respective scales, and η is the unknown shift of the origin. Writing

$$Z = \frac{\sigma(y + \eta)}{\sqrt{x}} \tag{82}$$

we obtain from equations (71), (80)–(82)

$$\beta = \frac{2}{\sqrt{\pi}} \frac{\sqrt{m_t}}{m_E} \frac{E_0^3}{\alpha F R_{02}} \frac{1}{Z}. \tag{83}$$

The quantity Z, a constant for each experiment, is best established as follows: For a series of selected values of x_i find the corresponding y_i by measuring the oscilloscope trace. In the vicinity of the origin, choose a value of x_1 at point where the corresponding y_1 can be ascertained with reliability. For the remaining pairs of x_i, y_i the desired Z_i follows from equation (82) in the form

$$Z_i = \frac{y_i - y_1}{[\sqrt{(x_i)}/\sigma_i] - [\sqrt{(x_1)}/\sigma_1]} \tag{84}$$

where the magnitudes of σ and σ_1 are given by equations (69) and (81), respectively. Because of errors in measurement, the various values of Z_i differ somewhat one from another; the most probable value is given by equation

$$Z = \frac{1}{n} \sum_{i=2}^{n+1} Z_i \tag{85}$$

where n is the number of values of Z_i. When the oscilloscope trace is measured with a profile projector, the limit relative error of Z is usually $\bar\varkappa_z \leqslant 0{\cdot}01$ for $n \geqslant 8$ [10]. The limit relative errors of the remaining quantities appearing in equation (83) can be estimated with reserve as follows: $\bar\varkappa_{m_t} = 0{\cdot}02$; $\bar\varkappa_{m_E} = 0{\cdot}01$; $\bar\varkappa_{E_0} = 0{\cdot}005$; $\bar\varkappa_\alpha = 0{\cdot}01$; $\bar\varkappa_F = 0{\cdot}01$; $\bar\varkappa_{R_{02}} = 0{\cdot}01$. The limit relative error of β is then obtained from relation

$$\varkappa_\beta^2 = \tfrac{1}{4}\bar\varkappa_{m_t}^2 + \bar\varkappa_{m_E}^2 + 9\bar\varkappa_{E_0}^2 + \varkappa_\alpha^2 + \bar\varkappa_F^2 + \varkappa_{R_{02}}^2 + \bar\varkappa_Z^2, \tag{86}$$

whence follows

$$\varkappa_\beta = 0{\cdot}03.$$

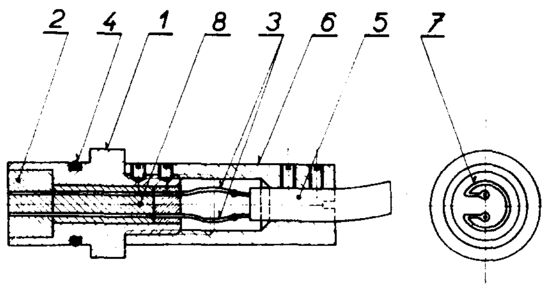

FIG. 4. Thin-film resistance thermometer.
1, sleeve; 2, glass plate; 3, platinum leads; 4, sealing ring; 5, h.f. cable; 6, shield; 7, nickel film; 8, ceramic bicapillary.

Table 5. Results of check measurement of β

Pickup	F (mm^2)	U (mm)	$\alpha \cdot 10^3$ (1/degC)	β^*	β (kg/s$^{5/2}$ degC)	β_1	$\delta\beta$	$\delta\beta_1$
No. 21	15·9	73·5	2·325	1113	1118	1191	0·004	0·065
No. 23	31·6	68·2	1·965	1113	1143	1167	0·026	0·046

The same error can also be expected in the case where β is being determined from the measurements of λ_0, ρ_0 and c_0.

Tests intended to verify our conclusions arrived at in the foregoing discussion, were carried out on glass. Samples of the glass were measured for thermal conductivity, density and specific heat using the steady-state methods. As the measurements indicate, the samples are at 20°C characterized (within the limits of error stated above) by a value of $\beta^* = 1113$ kg/s$^{5/2}$ degC. Pickups (Fig. 4) prepared from the measured samples were vacuum coated with a resistance film of nickel. Pickups Nos. 21 and 23 whose characteristic values, established by measurement, are listed in Table 5, were used in subsequent tests.

A series of experiments conducted at 20°C, was undertaken with these pickups using the method described above. The mean values of β obtained in these experiments are listed in Table 5 together with deviations

$$\delta\beta = \frac{\beta - \beta^*}{\beta}. \tag{87}$$

The deviations lie within the limits of estimated error of measurement; this means that all the effects that might have come into play, have been accounted for by the method. The results of measurement have enabled us to judge whether or not we have been justified in introducing the correction σ dependent on time. If such a correction factor is justified, the values of Z_t established for $\sigma(t)$ from the various points of the oscilloscope trace, should appear as unsystematic deviations whereas the deviations of Z_t determined for $\sigma =$ const. (e.g. $\sigma = 1$) should be systematic. Figure 5 in which the results of one of the experiments are plotted, gives evidence that it was so. The dependence of

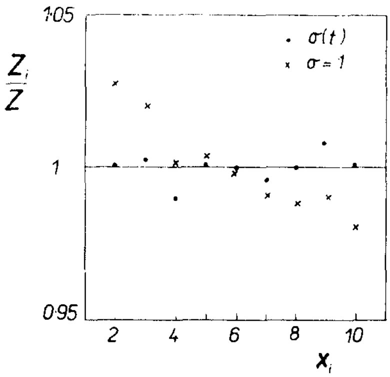

FIG. 5. Deviations of values of Z_t.

$\sigma(t)$ is mostly due to the last term on the right-hand side of equation (69). Neglecting this term would result in values β_1 with the corresponding deviations $\delta\beta_1$ given by equation (87). These values are listed in Table 5, and it is clear that deviations $\delta\beta_1$ exceed both the deviations $\delta\beta$ and the estimated error of measurement $\bar{\varkappa}_\beta$. The last term on the right-hand side of equation (69) expresses the effect of heat flow parallel to the wall surface; as our exposition has indicated, this effect must be taken into consideration if we wish to enhance the accuracy of the method.

MEASURING THE TEMPERATURE RISE OF THE WALL SURFACE

Another prerequisite for the application of equations (10) and (29) is a measurement of the time variation of temperature rise of the wall surface that would enable us to determine the coefficients in relations (3) and (13) with sufficient accuracy. Thin-film resistance thermometers are conveniently used for this purpose in conjunction with the bridge illustrated in Fig. 6. Prior to the beginning of the process being studied, the bridge is balanced with potentiometers R_4, R_5

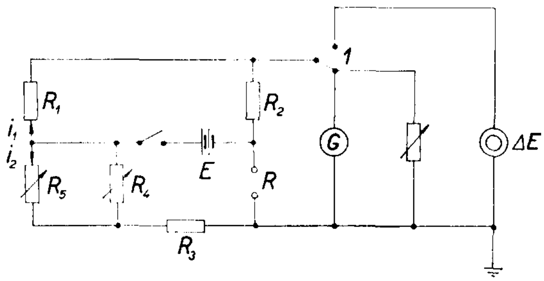

FIG. 6. Bridge for measuring temperature differences. $R_1 = R_2 = 400\ \Omega$; $R_3 = 47\ \Omega$; $R_4 = 100\ \Omega$; $R_5 = 10\ \Omega$.

and connected through switch 1 to an oscilloscope which displays the variation of potential difference ΔE between the nodal points in time t. This bridge is simpler than the one schematically shown in Fig. 2 because of less stringent demands made on its function (it is, for example, not necessary to compensate for the change in heat flow).

An analysis of the electric conditions of the bridge (Fig. 6) indicates that

$$i_2 = \frac{E}{R + R_x} \tag{88}$$

$$\Delta E = E\frac{R_1R - R_2R_x}{(R + R_x)(R_1 + R_2)} \tag{89}$$

where R is the resistance of the thin-film thermometer, E the voltage of the source, and R_x the resultant value of resistances R_3, R_4, R_5.

It holds for a bridge balanced prior to the measurement that

$$R = R_0; \quad \frac{R_x}{R_0} = \frac{R_1}{R_2} \tag{90}$$

hence equations (42), (89) and (90) give

$$T = \frac{1}{\alpha}\frac{\Delta E}{E}\frac{1 + (R_1/R_2)}{\dfrac{1}{1 + (R_2/R_1)} - (\Delta E/E)}. \tag{91}$$

An advantage of this procedure lies in that the measurement of the temperature rise is independent of the basic resistance R_0 of the thermometer. Equation (91) enables us to determine the ratio of R_1/R_2 so as to yield the maximum ΔE for the given values of α, E, T. The ratio

$$\frac{R_1}{R_2} = \sqrt{(1 + \alpha T)} \tag{92}$$

and since αT is always small compared with unity, the highest sensitivity is attained with resistances R_1 and R_2 approximately equal (Fig. 6).

The choice of voltage E is another question. The higher the voltage, the more sensitive the bridge but the higher the temperature rise of the film due to Joule's heat. Since such a temperature rise might constitute a source of error, there must exist an optimum voltage at which the accuracy of measurement will be highest. The effect of Joule's heat can be expressed with sufficient accuracy as

$$R_0 i_2^2 \cong \Lambda F T_0 \tag{93}$$

where Λ is the heat-transfer coefficient and T_0 the respective temperature rise of the film. The latter is obtained approximately from equations (88), (90), (92) and (93) in the form

$$T_0 \cong \frac{E^2}{4R_0\Lambda F}. \tag{94}$$

To compute the errors, equations (91) and (92) serve to express the approximate value of the measured temperature rise

$$T \cong \frac{4}{\alpha}\frac{\Delta E}{E} \tag{95}$$

The values of ΔE are established by evaluating the respective oscillograms and since there are no initial transient phenomena present, we may put $\eta = \theta$ in equation (80). Estimating the absolute error brought about by Joule's heat, with the aid of the relation

$$\varkappa_{T_0} = \psi T_0 \tag{96}$$

we may write the relative error of the measured temperature rise T on the basis of equations (80) and (94)–(96) in the form

$$\varkappa_T^2 = \bar{\varkappa}_\alpha^2 + \varkappa_{m_E}^2 + \varkappa_E^2 + \left(\frac{4m_E}{\alpha E T}\varkappa_y\right)^2 + \left(\frac{\psi E^2}{4R_0F\Lambda T}\right)^2 \tag{97}$$

where $\varkappa_y$ is the absolute error of measurement of co-ordinates y on the oscilloscope trace.

Voltage E at which error $\bar{\varkappa}_T$ will be a minimum, follows from equation (97) in the form

$$E = 2\sqrt[3]{\left(\frac{\sqrt{2}}{\alpha\psi} m_E \varkappa_y R_0 F \Lambda\right)}. \tag{98}$$

To evaluate relations (97) and (98), it is first necessary to establish the value of Λ. The pertinent measurements were made on pickups designed in accordance with Fig. 4 built in the steel wall of a shock tube with their resistance film in the vertical plane. During the measurements, the air in the shock tube was kept at atmospheric pressure and 20°C, and the dependence of film resistance on heating current i_2 measured. According to the results, Λ is virtually independent of T_0 and attains a value of $\Lambda \cong 1000$ W/m² deg C. A scatter of up to ±20% exhibited by the various pickups, is due to the different clearances with which the pickups were built in the wall. The share of natural convection in the value of Λ was indicated as about 1%; hence neither the position of the resistance film nor the pressure in the measuring space will exert perceptible effect on the value of Λ.

For the purpose of computation, we have estimated $\bar{\varkappa}_\alpha = 0{\cdot}01$, $\bar{\varkappa}_{m_E} = 0{\cdot}01$, $\bar{\varkappa}_E = 0{\cdot}005$, $\varkappa_y = 3 \times 10^{-4}$ m, $\psi = 0{\cdot}1$, taken $\alpha = 2{\cdot}7 \times 10^{-3}$ 1/°C, $m_E = 0{\cdot}1$ V/m, $\Lambda = 1000$ W/m² deg C and assumed $R_0 = 65\ \Omega$ in equation (97). The results are presented in Figs. 7 and 8. It is clear that a temperature rise T 3 degC $\geqslant$ can be measured with a limit relative error of about 2%, and that larger areas of the films are more convenient, $F \geqslant 10$ mm² being already satisfactory for the purpose.

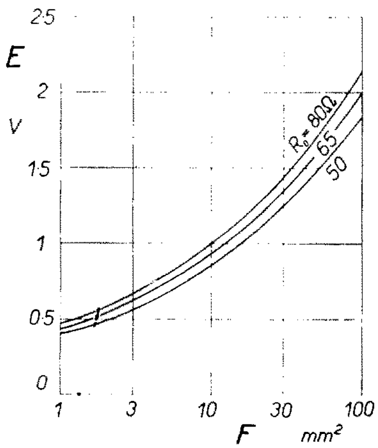

FIG. 7. Suitable energizing voltage of bridge from Fig. 6.

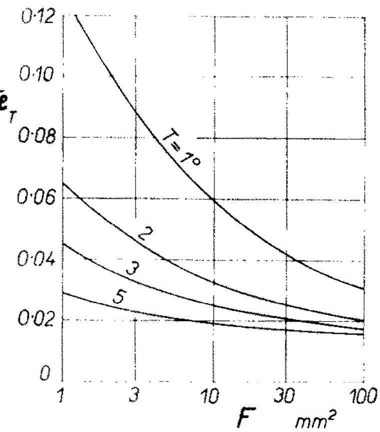

FIG. 8. Error of measurement of temperature differences effected with thin-film thermometer and bridge from Fig. 6.

CONCLUSION

Taking into consideration relations (3) and (13), we are in a position to state that the relative errors of expressions in the braces of equations (10) and (29) are of the same order of magnitude as the relative errors of the measured temperature rise of the surface. The limit relative error of heat flow q determined through the use of the method discussed in the paper, can be expressed in the form

$$\varkappa_q^2 = \bar{\varkappa}_T^2 + \bar{\varkappa}_\beta^2 + \tfrac{1}{4}\varkappa_t^2. \tag{99}$$

Since the error burdening the measurement of time t is preponderantly given by the error of scale m_t ($\varkappa_t = \bar{\varkappa}_{m_t}$), and the temperature rise usually studied in the measurement of unsteady heat flows, exceeds 3 degC, we can—in view of the preceding explanation—estimate with qualifications the limit relative errors as follows: $\bar{\varkappa}_T = 0{\cdot}02$, $\bar{\varkappa}_\beta = 0{\cdot}03$, $\bar{\varkappa}_t = 0{\cdot}02$. Equation (99) then gives $\bar{\varkappa}_q = 0{\cdot}04$. Since Hall and Hertzberg [1] estimate the error of measurement of unsteady heat flows with thin-film resistance thermometers as ±5 to 15%, it can be stated that the elaboration of the method has resulted in increased accuracy. Moreover, equations (10) and (29) enable us to carry out the determination of the heat flow from the data of thin-film thermometers in a substantially simpler way in many instances.

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Résumé—Dans cet article, les équations pour le calcul du flux de chaleur instationnaire à une paroi sont obtenues dans le cas où la variation dans le temps de la température à la surface de la paroi peut s'exprimer par une puissance ou une série trigonométrique. Pour utiliser ces équations, il est d'abord nécessaire de connaître d'une façon sûre la racine carrée du produit de la conductivité thermique, de la densité et de la chaleur spécifique du matériau de la paroi, désignée par β, et de mesurer la variation de l'augmentation de la température de surface avec suffisamment de précision.

Les deux opérations peuvent être accomplies en employant des thermomètres à résistance à film mince consistant en une mince couche métallique déposée sur une paroi isolante électriquement. L'article expose d'une façon détaillée la méthode instationnaire de la mesure de β qui facilite la détermination de cette valeur avec une erreur d'environ 3%. Une méthode de mesure de l'augmentation de la température de surface dépassant 3 degC avec une erreur d'environ 2% est également décrite. La méthode exposée dans l'article permet de déterminer le flux de chaleur instationnaire à la paroi avec une erreur d'à peu prés 4%.

Zusammenfassung—Zur Berechnung des instationären Wärmestromes an eine Wand werden Gleichungen für die Fälle agegeben in denen die zeitliche Änderung der Oberflächentemperatur der Wand als Potenzreihe oder trigonometrische Reihe ausgedrückt werden kann. Zum Gerbrauch dieser Gleichungen ist es notwendig zuverlässig die Quadratwurzel aus dem Produkt von Wärmeleitvermögen Dichte und spezifischer Wärme des Wandmaterials, als β bezeichnet, zu bestimmen und die zeitliche Änderung der Oberflächentemperaturerhöhung genügend genau zu messen. Beide Aufgaben können mit einem Dünnfilmwiderstandsthermometer das aus einem dünnen metallischen Belag auf einer elektrisch isolierenden Wand besteht gelöst werden.

Es wird eine genaue Ausarbeitung der instationären Methode der β-Messung gegeben; sie ermöglicht die Bestimmung dieses Wertes mit einem Fehler von etwa 3 %. Eine Methode zur Messung der Temperaturerhöhung der Oberfläche bei mehr als 3 grd und einem Fehler von etwa 2 % wird ebenfalls beschrieben. Das in der Arbeit diskutierte Verfahren erlaubt die Bestimmung des instationären Wärmestromes an eine Wand mit einem Fehler von etwa 4 %.

Аннотация—В статье выводятся уравнения для расчета нестационарного теплового потока на стенке для случаев, когда температурные изменения во времени на поверхности стенки можно опистаь с помощью степенного или тригонометрического ряда. Для использования этих уравнений необходимо установить надежность значения квадратного корня произведения теплопроводности, плотности и удельной теплоемкости материала стенки, обозначаемого через β, а также с достаточной точностью измерить изменения во времени температуры на поверхности стенки. Этого можно достичь,

используя пленочные термометры сопротивления, состоящие из тонкого металлического слоя, нанесенного на поверхность стенки из диэлектрика. В статье детально излагается нестационарный метод измерения β, который дает возможность определить эту величину с точностью до 3%. Описан также метод измерения увеличения температуры на поверхности, превышающего 3°с точностью около 2%. Изложенная в статье методика позволяет определить нестационарный тепловой поток на стенке с ошибкой около 4%.